ISSN 2229-5518

A TIME SERIES ANALYSIS OF DAILY EXCHANGE RATE OF U.S DOLLAR TO NAIRA FROM 2016-2017 (RECESSION PERIOD)

BY
Chrysogonus Chinagorom Nwaigwe, Godswill Uchenna Atuzie, Iheanyi Chinedu Obinwanne, Chukwudi Justin Ogbonna
Department of Statistics, Federal University of Technology Owerri, Imo State
Correspondence: chrysogonusnwaigwe@yahoo.com

ABSTRACT

The appropriate time series model of the official daily exchange rate of Nigerian Naira for US Dollar in terms of buying rate, central rate and selling rate, from the period of January 1, 2016 to May 19, 2017 (recession period) was investigated. Box and Jenkins approach was applied for the modelling of naira/dollar daily exchange rate using ARIMA model. The results of the analysis show that the series became stationary after first differencing. Based on Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC), the best model that explained the series was found to be ARIMA (0, 1, 1) for the three series. The model was also confirmed through diagnostic checking. The error was found to be white noise and there was no presence of serial correlation. A forecast for period of 182 days was made which indicated that the naira will continue to depreciate within the forecast period. The performance of the three models ARIMA (0, 1, 1) in terms of buying rate, central rate, and selling rate for out-of-sample also show that ARIMA (0, 1, 1) for selling rate and the buying rate model outperformed the central rate model in the three exchange rate series for the period forecast. The performance of the three ARIMA (0, 1, 1) models for buying rate, central rate and selling rate revealed that the selling rate model had the least ME, MSE, RMSE and MAPE.

Key Words: Exchange Rates, Nigerian Naira, U. S. Dollar, Time Series Modeling, Forecast

1. Introduction

Time series is the record of outcomes of a variable according to time. The outcomes may be recorded daily, weekly, monthly, quarterly, yearly or at any other specified interval of time. It is known that time series data are volatile. Undoubtedly, daily exchange rate of one currency for another currency forms a time series. A time series may be studied in order to obtain an appropriate model which may be necessary for planning purpose. For example, having an idea of the future exchange rate of a currency for another currency could help the country to guide against inflation, determine its balance of payments and formulate feasible economic policies, among other benefits.

Research works on modelling of exchange rate of a currency for another currency abound. David, Hussaini and Gulumbe (2016) examined the exchange rate of naira for other four currencies using the generalized autoregressive conditional heteroscedasticity (GARCH) and its asymmetric variance. Results showed that three of the four return series indicated that heteroscedasticity was present while the asymmetric model indicated different impacts for both negative and positive shocks. According to the paper, results obtained from the asymmetric model showed superior forecasting performance to the symmetric GARCH. Oletunji and Bello (2013) carried out a time series modeling of the official exchange rate of Nigeria Naira for US dollar using exchange rate data from 2000 – 2012. ARIMA (1, 1, 2) was confirmed to be the appropriate model for the series. Tasi’ u (2014) carried out a time series modeling of exchange rates of currencies of nine selected countries (including Nigeria) for U.S. dollar with multivariate GARCH models. The data used covered from January, 1999 to February 2014.

Emenike and Ali (2016) modelled the behavior of Naira/Us dollar exchange rates in Nigeria in order to find out if the series follow autoregressive conditional heteroscedasticity (ARCH). They used monthly data from January 2000 to December 2013. Results obtained showed that there is presence of heteroscedasticity which showed the appropriateness of the ARCH family models for modelling the volatility.

Onasanya and Adeniji (2013) used the fundamental (box and Jenkins) approach for the modelling of the exchange rate of Nigeria naira for US dollar for the period January, 1994 to December, 2011. The results of the study showed that there is an upward trend and the second difference of the series was stationary. Based on the AIC and BIC criteria, the best model that explained the series was found to be

ARIMA (1, 2, 1). A forecast of twelve months' period from the fitted model showed that the naira will continue to depreciate.

Mohammed and Abdulmuahymin (2016) investigated the exchange strength of Nigeria Naira with respect to United states dollar using data from the period 1972 to 2014. They found out that the appropriate model was ARIMA (0, 2, 1). Awogbeni and Alagbe (2011) using GARCH model examined the volatility of the exchange rate of Nigeria naira for UK pound sterling on the other hand using GARCH model. A monthly average rate data from 2007 to 2010 were used for the study. The result of normality test indicated that the series residuals are asymmetric. GARCH (1, 1) was identified as the appropriate model for the variation in the series.

In the present paper, the official daily exchange rates of Nigeria Naira for US Dollar in terms of buying rate, central rate and selling rate, from the period of January 1, 2016 to May 19, 2017 (recession period) are studied independently using a time series model. This seems to be the first time that the three rates are studied simultaneously. Previous works had concentrated on only one of the rates. Also, no work on time series modeling of exchange rate of Nigerian Naira to U. S. Dollar has been seen to cover the stated recession period which is of immense significance in the study of the series.

2.**Materials and Methodology**

In this section, the materials that were used for the study are stated and the statistical techniques that were employed in order to achieve the objectives of the research are discussed.

2.1 Materials

The data set is the Nigerian official daily exchange rate of Naira against US dollar, which was obtained from the Central Bank of Nigeria (CBN) categorized in terms of buying rate, central rate and selling rate, from the period of January 4, 2016 to May 19, 2017.

2.2 Methodology

In time series, changes are usually attributable to some factors. These factors are called components of the time series. In this paper, the graphical method was used to identify the components of the time series.

The Box and Jenkins approach was employed for the modeling of the series. This approach uses integrated autoregressive, moving average, known as ARIMA. The first step in developing a Box-jenkins model is to determine if the time series is stationary and if there is any significant seasonality that needs to be modelled (Box and Jenkins, 1970). A time series is said to be stationary if there is no systematic trend in mean (no trend) and variance and its periodic variations have been removed, this implies that, the expected value of the time series does not depend on time and the auto-covariance function.

For a stationary process,

$$\mu = E\left(X_t\right) = 0 \tag{2.1}$$

$$Var\left(X_t\right) = Var\left(X_{t+k}\right) = R\left(0\right) = 1 \tag{2.2}$$

In this paper, the graphical method, Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP) were used to test for the stationarity of the series. The second step is the identification of the appropriate ARIMA models through the study of the autocorrelation and partial autocorrelation functions. The next step is the estimation of the parameters of the ARIMA model chosen. After identification of the models, diagnostic checking of the model is performed. Ljung Box test are used for the model adequacy check. If the model is not adequate, stage one is repeated to identify an alternative model but if it is adequate, then the final stage of the process is proceeded to. The next step is using the selected model for forecast and the process ends.

When a time series is not stationary, it may be differenced d number of times in order to achieve stationarity. Such a time series is said to be integrated denoted as $I\left(d\right)$. Differencing may be used to simplify the structure of a time series that is non-stationary, thus, forming a new series. First-order differencing removes a linear trend. For example,

$$\nabla x_t = x_t - x_{t-1} \tag{2.3}$$

First-order seasonal differencing removes a fixed seasonal pattern.

$$\nabla_s x_t = x_t - x_{t-s} \tag{2.4}$$

It is possible for differencing to be applied repeatedly to a series, thus, giving

$$W_t = \nabla^d \nabla_x^D x_t \tag{2.5}$$

where d and D are the orders of differencing.

There are three classes of linear models which are most useful in modeling the dependence between stationary series.

2.2.1 Autoregressive (AR) models

In these models, the current value of the series, can be explained as a function of p past values, $x_{t-2}...x_{t-p}$, where p determines the number of steps into the past needed to forecast the current value. An x_{t-1} autoregressive model of order p, $\left[AR(p)\right]$ could be explicitly written as,

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + w_t \quad (2.6)$$

where, x_t is stationary series, $\phi_1, \phi_2, ..., \phi_p$ are the parameters of the $AR(\phi_p \neq 0)$; w_t is a Gaussian white noise series with mean zero and variance $\sigma < \infty$, unless stated otherwise. The highest order p is referred to as the order of the model.

2.2.2 Moving – Average MA(q) Model

A time series (Y_t) is said to follow a Moving Average process if its current values depend on its past shocks. This implies that, the forecast values of the series depend on the past errors. Thus, a Moving Average process of order q, $\left[MA(q)\right]$ is given as,

$$Y_t = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t, \quad t = 1, 2, ... \quad (2.7)$$

Where $\theta_1, ..., \theta_q$ are parameters, μ is the expectation of Y_t and ε_t is white noise (error term).

2.2.3 ARMA (p, q) Model

The notation $ARMA(p,q)$ refers to the model with p autoregressive terms and q moving- average terms.

Equations 2.6 and 2.7 give the $ARMA(p,q)$ model, given by,

$$Y_t = \lambda + \sum_{i=1}^{p} \varphi_i + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (2.8)$$

2.2.4 Model Identification

This involves examining a series by various methods to find out which of the class of linear process appears to be most appropriate. This means determining the values of p, q, and d. This is done by observing the graph of the data or autocorrelation, or partial autocorrelation functions (Makridakis et al., 1998). The

ISSN 2229-5518

following table summarizes the behavior of the theoretical ACF and $PACF$ of the $ARMA$ models (Esam, 2017).

Table 2.1 behavior of the theoretical ACF and PACF of the ARMA models

	ACF	**PACF**
White noise	All zeros	All zeros
$AR(p)$	Tails off as exponential decay.	Cuts off after p lags
$MA(p)$	Cuts off after q lags	Tails off as exponential decay.
$ARMA(p,q)$	Tail off after lag $(q-p)$ (sometimes in an oscillating manner)	Tail off after lag $(p-q)$ (sometimes in an oscillating manner)
Random walk	No decay to zero	All zeros after lag 1

2.2.5 Best Model Selection Criteria

When fitting models, there is the tendency that some models will be competing. Thus, it is appropriate to use a model selection criterion to select the most adequate model. In this study, the AIC and the BIC were employed to select the most adequate model. For a given data set, several competing models may be ranked according to their AIC or BIC values, the one having the lowest information criterion value is said to be the best model. The AIC and BIC are defined respectively as,

$$AIC(n) = 2k + n\log\left(\frac{RSS}{n}\right) \tag{2.9}$$

$$BIC(n) = \log\left(\sigma_e^2\right) + \frac{k}{n}\log(n) \tag{2.10}$$

where,

k is the number of parameters in the statistical model, RSS is the residual sum of squares of the estimated model, n is the number of observations in the data, σ_e^2 is the error variance (Akaike, 1974)

2.3 Estimation of Model parameters:

In this paper, the maximum likelihood method of estimation is employed to estimate the model parameters.

2.4 Test for Heteroscedasticity

To ensure that the fitted model is adequate, the assumption of constant variance must be achieved. In this paper, the **ARCH-LM test** is another procedure that uses a statistic that is used to test for the presence of conditional heteroscedasticity in the model residuals. The test procedure is,

H_0: Residuals are homoscedastic

H_1: Residuals are heteroscedastic

The test statistic is

$$LM = nR^2 \tag{2.11}$$

where n is the number of observations and R^2 is the coefficient of determination of the auxiliary residual regression;

$$\ell_t^2 = \beta_0 + \beta_1 \ell_{t-1}^2 + \beta_2 \ell_{t-2}^2 + \ldots + \beta_q \ell_{t-q}^2 + \mu_t \tag{2.12}$$

where ℓ_t is the residual.

Decision rule: Rejected H_0 if the p-value is less than the level of significance and conclude that there is heteroscedasticity.

2.5 Test for Serial Correlation

The Ljung-Box test was used for testing the assumption that the residuals contain no serial correlation up to any order k. The test procedure is,

H_0: There is no serial correlation up to order k
H_1: There is serial correlation up to order k:
The test statistic is given by;

$$Q_m = n(n+2)\sum_{k-1}^{m} \frac{\hat{\ell}^2(k)}{n-k} \sim \chi^2_{(m-p-q)} \tag{2.13}$$

where, n is the sample size, $\hat{\ell}^2(k)$ is the sample autocorrelation of the residual series at lag k, for $(p+q) < m < n$, and m is the number of lags being tested.

The decision rule is to reject H_0 if Q_m (or Q_m) $\geq \chi^2_{\alpha(m-p-q)}$ where $\chi^2_{\alpha(m-p-q)}$ denotes the $100(1-\alpha)th$ percentile of a chi-squared distribution with $m-p-q$ degrees of freedom. When the p-value associated with Q_m is large, the model is considered adequate else the whole estimation process has to start again in order to get the most adequate model (Esam 2017).

2.6 Forecasting

The ultimate objective of model building is to provide forecasts of future values. In producing the forecasting using the fitted model, it is assumed that the conditions under which the model was constructed would persist in the period for which forecasts were made (Nwogu and Iwu, 2010).

The autoregressive representation

$$Y_t = \sum_{u=1}^{\infty} \pi Y_{t-u} + \varepsilon_t \qquad (2.14)$$

suggests predicting the next observation beyond $Y_1, \ldots, Y_T$ using

$$\hat{Y}_{T+1} = \sum_{u=1}^{\infty} \hat{\pi}_u Y_{T+1-u} \qquad (3.24)$$

where the $\hat{\pi}$ are obtained by substituting the estimated parameters in place of the theoretical ones.

2.7 ARIMA Model Forecast Performance:

Forecast error is often computed in order to assess the performance of the model in forecasting future values of the series. Mean error (ME), mean squared error (MSE), root mean squared error (RMSE) and mean absolute percentage error (MAPE) were used in this paper to assess the forecast errors. The smaller the mean squared error, the better the performance (Oladejo and Abdullahi, 2013).

3. Application

This section presents the results which were obtained from the various analysis of the data.

3.1 Graphical presentation of the exchange rate time series data

Figure (3.1, 3.2, and 3.3) below represent the time series plots, for buying rate, central rate and selling rate. The plots give an idea of the variability of the series and any component present in the series.

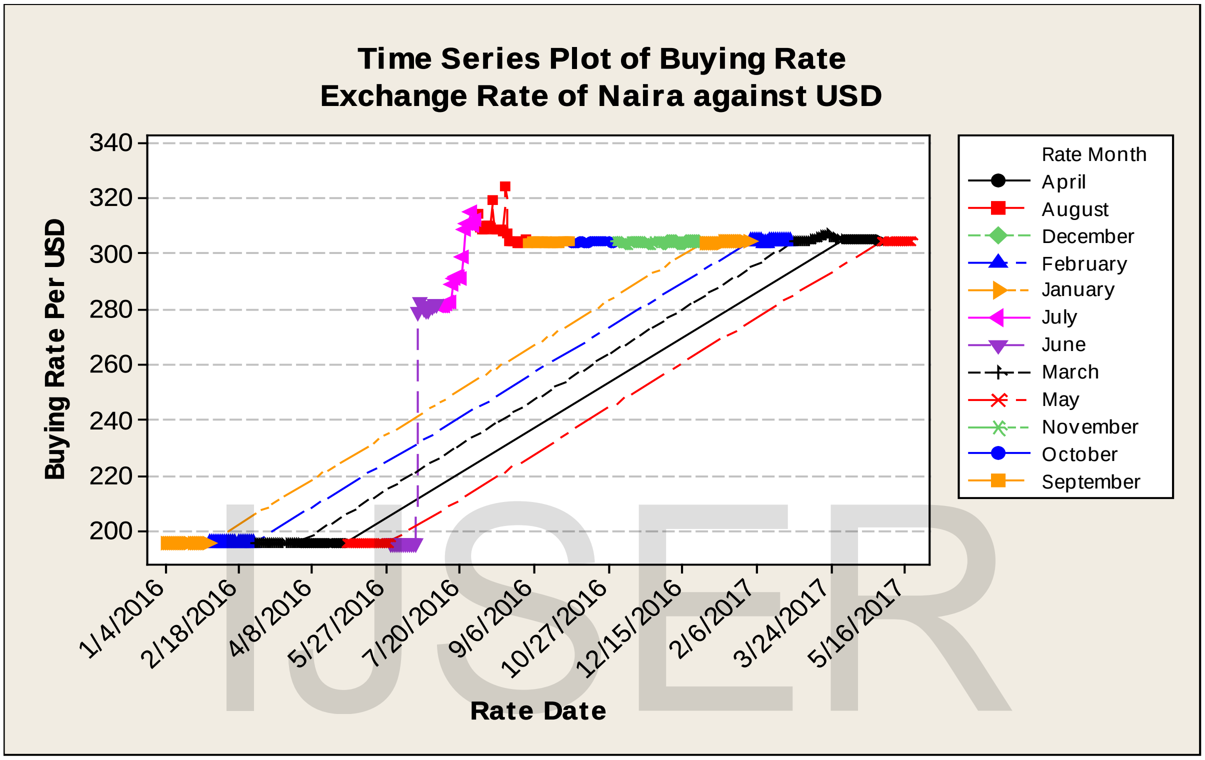


Figure 3.1: Time Plot of Buying Rate of Daily Exchange Rate of Naira to U. S. Dollar from January 2016 to May 2017.

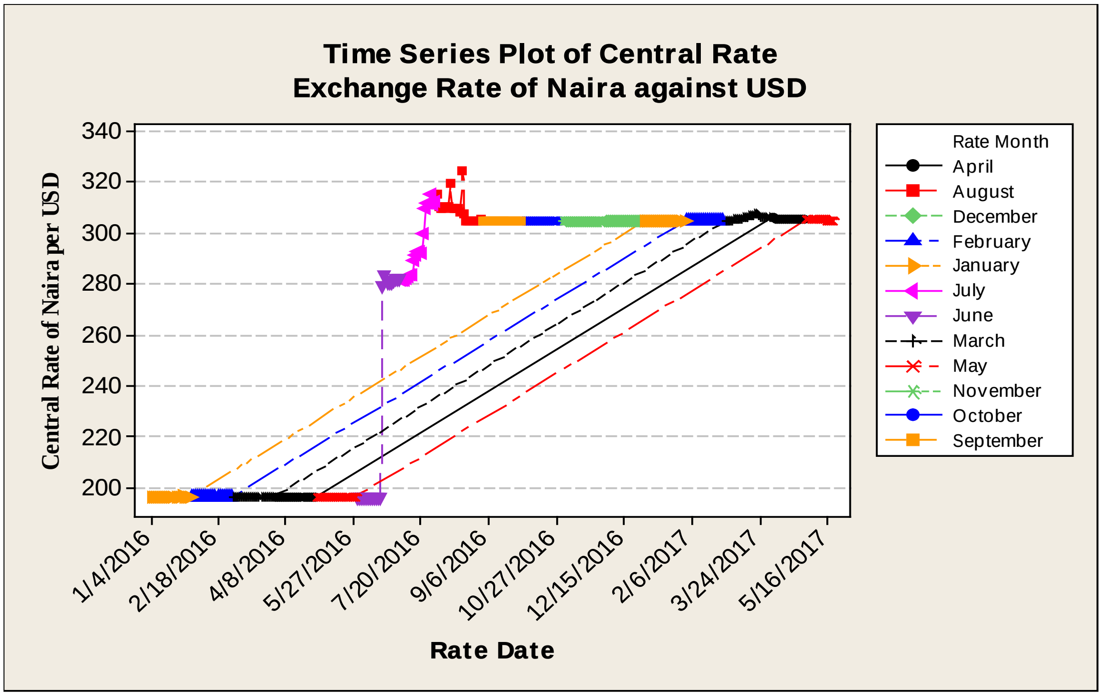


Figure 3.2: Time Plot of Central Rate of Daily Exchange Rate of Naira to us Dollar from January 2016 to May 2017.

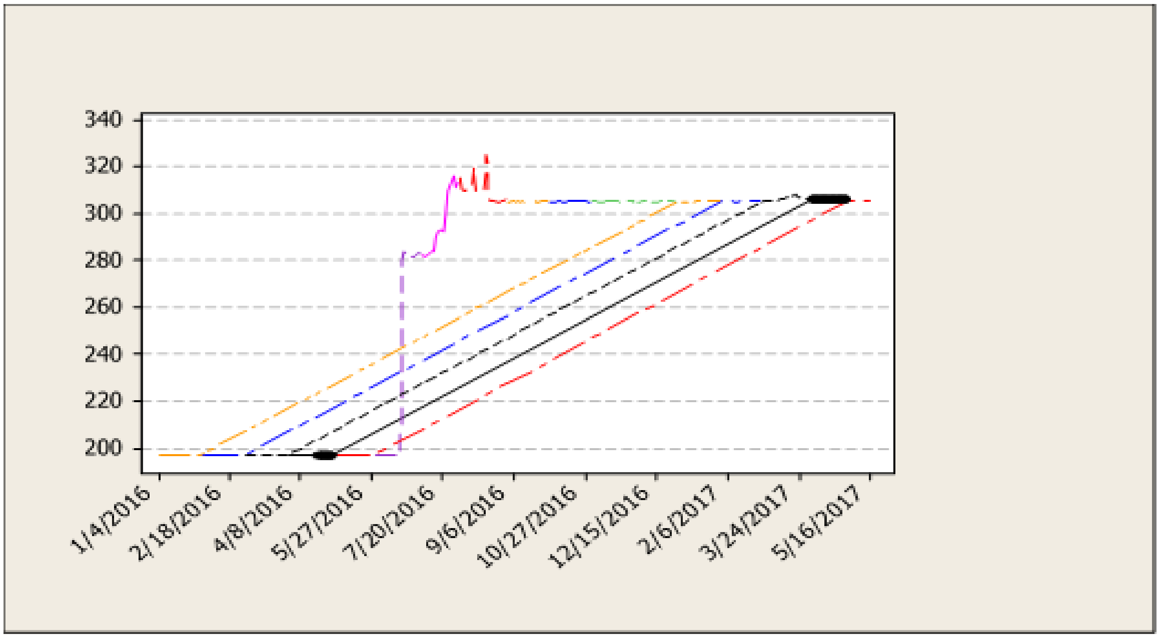


Figure 3.3: Time plot of Selling Rate of Daily Exchange Rate of Naira to U. S. Dollar from January 2016 to May 2017.

The results in Figures 3.1, 3.2 and 3.3 show that seasonal and cyclical components characterized the buying, central and selling rates of daily exchange rate of Nigeria to U. S. dollar.

The series were tested for stationarity using Dick-Fuller and Phillips tests and the following results were obtained.

Table 3.1 Results of Dick-Fuller Test

Value	Buying Rate	Central Rate	Selling Rate
Observed Value	-1.319	-1.322	-1.322
Critical Value	-3.416	-3.396	3.396
p-value	0.873	0.870	0.870

$\alpha = 0.05$

Table 3.2: Results of Phillips-Perron Test

Value	Buying Rate	Central Rate	Selling Rate
Observed Value	0.981	-1.322	0.983
Critical Value	-1.942	3.396	-1.942
p-value	0.914	0.870	0.914

$\alpha = 0.05$

The results of the Dick-Fuller Test and Phillips-Perron Test for buying, central and selling rates all show that the series are non-stationary because, in each case, the P-Value was greater than the level of significance.

Thus, the series were differenced in order to achieve stationarity. Figures 3.4, 3.5 and 3.6 below show the graphs of differenced series. It can be observed from the figures that stationarity of the series could be achieved through first-order differencing. This was supported by the results of tests for stationarity for the differenced series as shown tables 3.3 and 3.4 where for each test on each of the series, the p-value was greater than the α value 0.05.

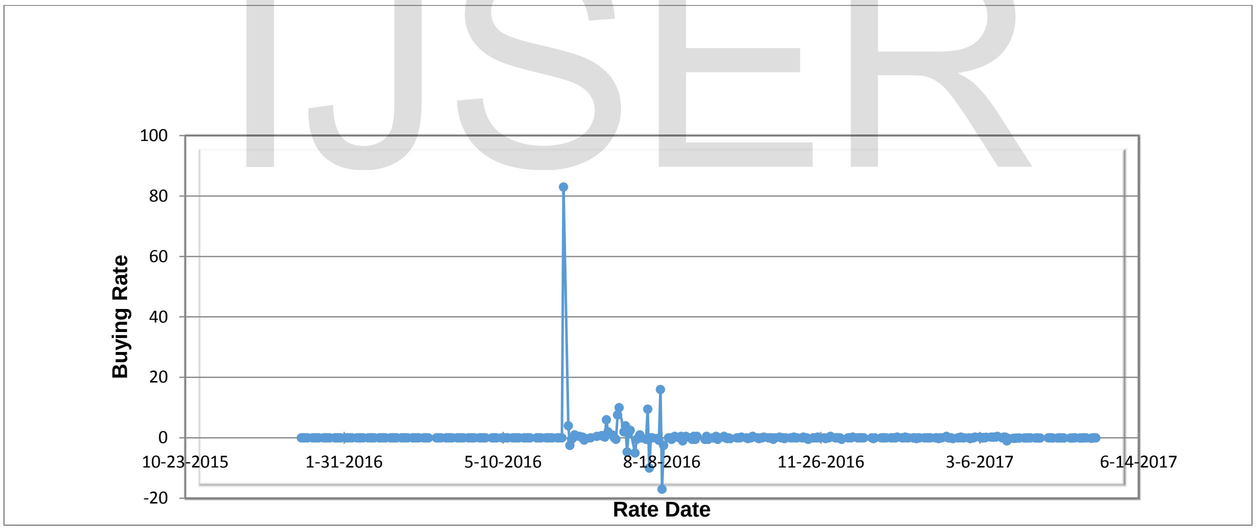


Figure 3.4: Plot of differenced Buying Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017

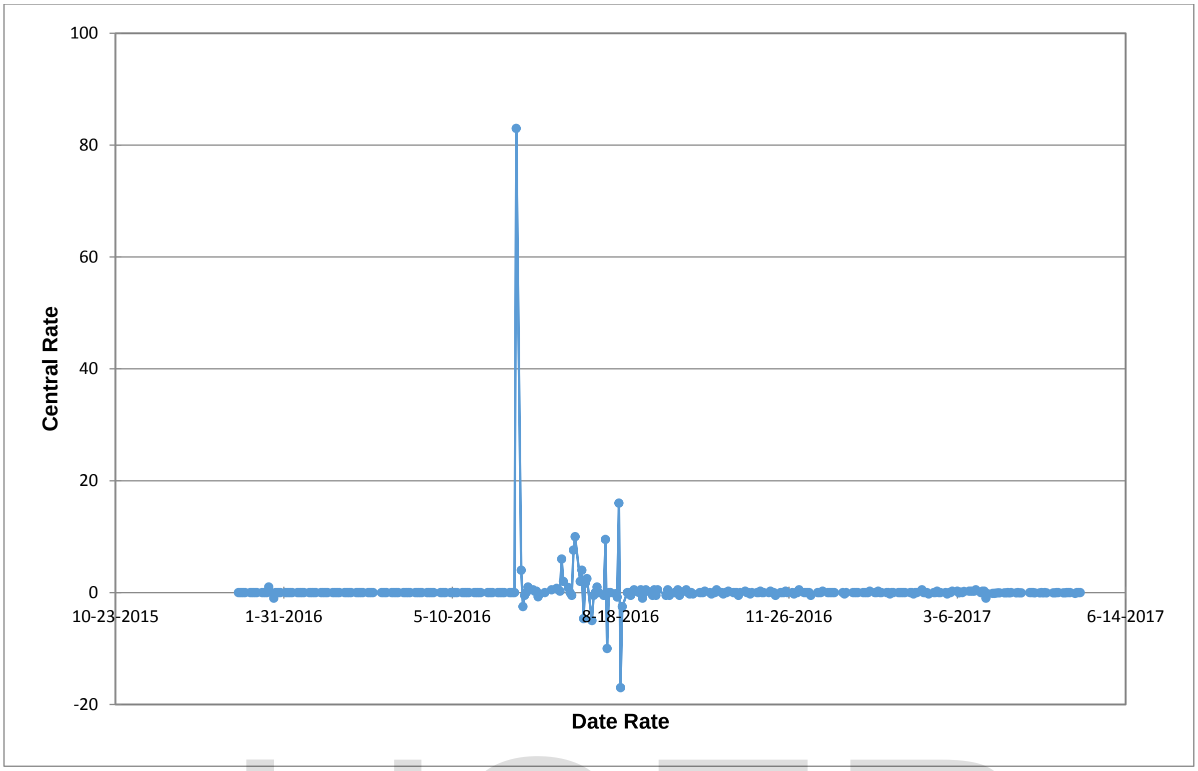


Figure 3.5: Plot of differenced Central Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017

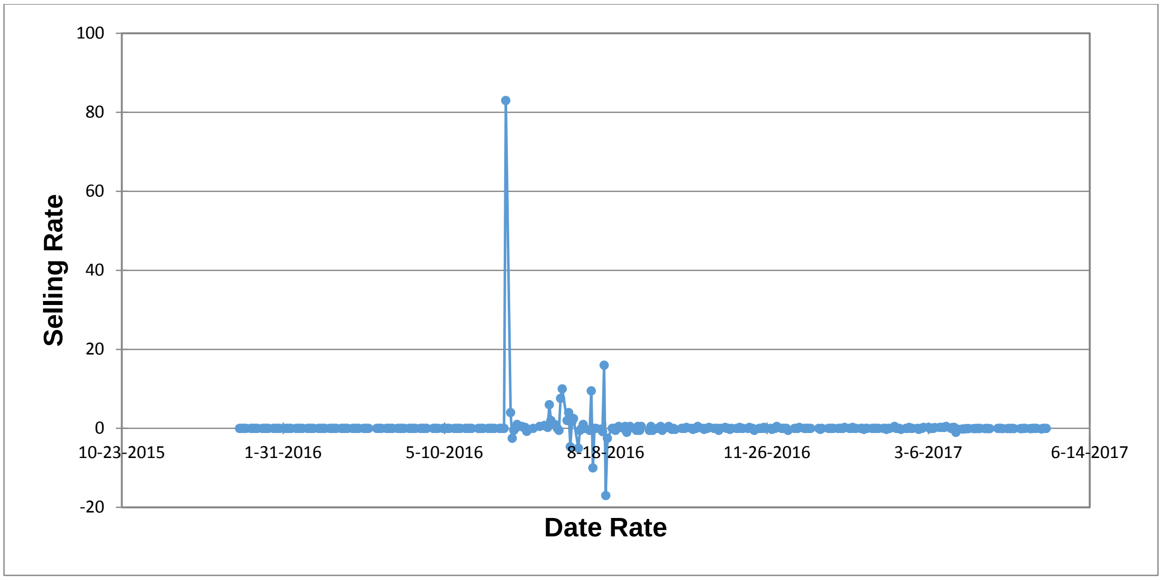


Figure 3.6: Plot of differenced Selling Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017

Table 3.3: Results of Dick-Fuller Test for Differenced Series

Value	Buying Rate	Central Rate	Selling Rate
Observed Value	-6.836	-6.836	-6.836
Critical Value	-3.416	-3.416	-3.416
p-value	<0.0001	<0.0001	<0.0001

α = 0.05

Table 3.4: Results of Phillips-Perron Test for Differenced Series

Value	Buying Rate	Central Rate	Selling Rate
Observed Value	-18.290	-18.292	-18.290
Critical Value	-1.942	-1.942	-1.942
p-value	<0.0001	<0.0001	<0.0001

α = 0.05

3.2 Application of Techniques for Model Identification

In this section, we determine the values of p, q and d for each of the series using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF).

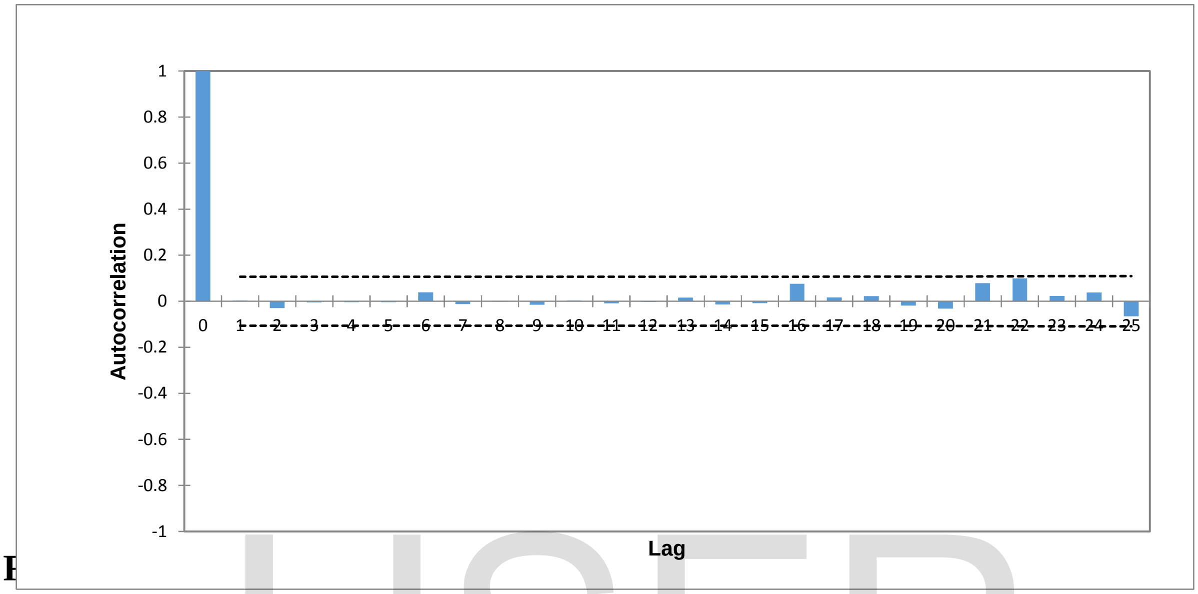


Figure 3.7: First differenced ACF Plot of Buying Rate of Daily Exchange rate of Naira to US Dollar from January 2016 to May 2017.

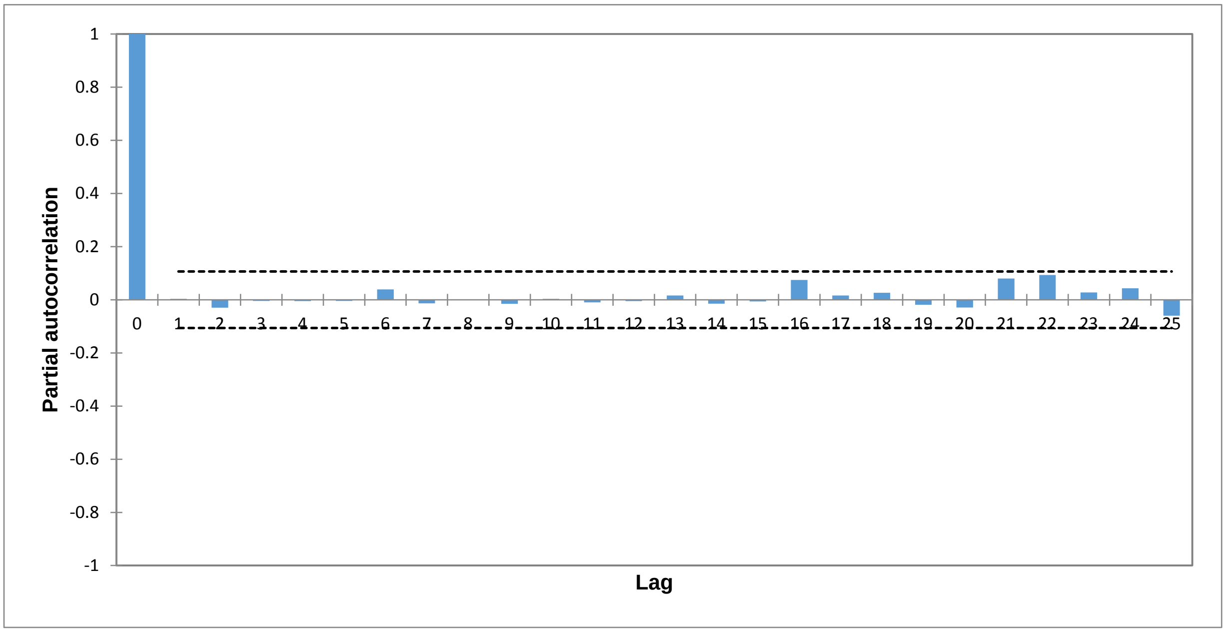


Figure 3.8: First differenced PACF Plot of Buying Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017.

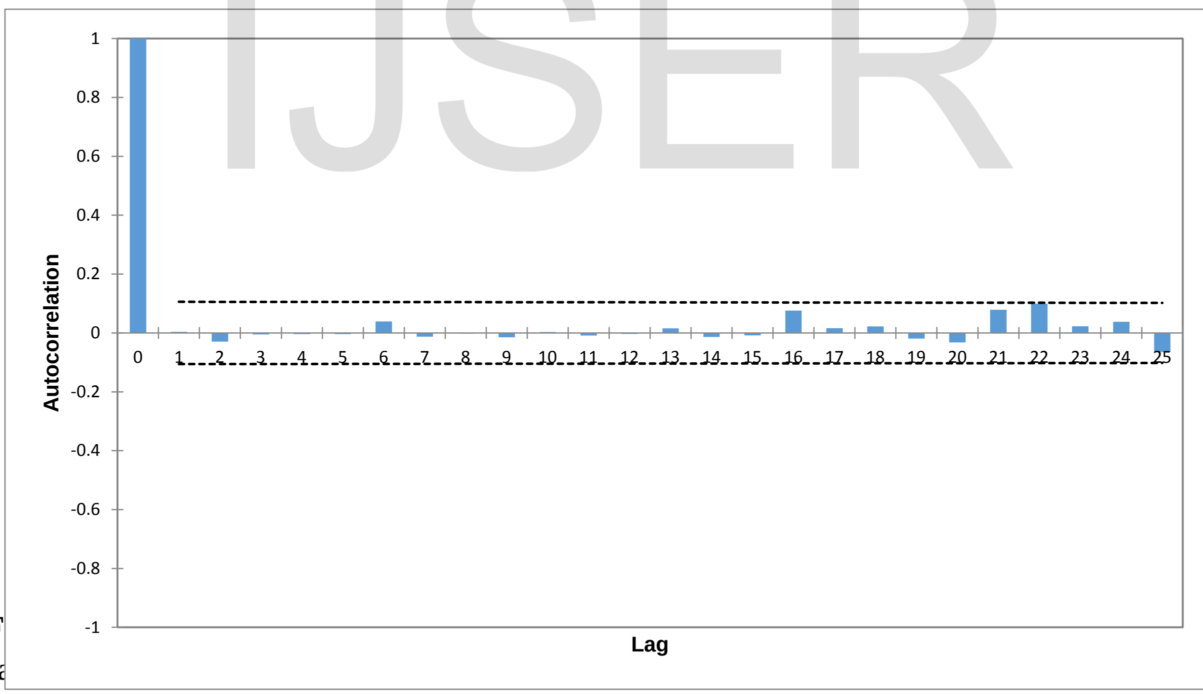


Figure 3.9: First differenced ACF Plot of Central Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017.

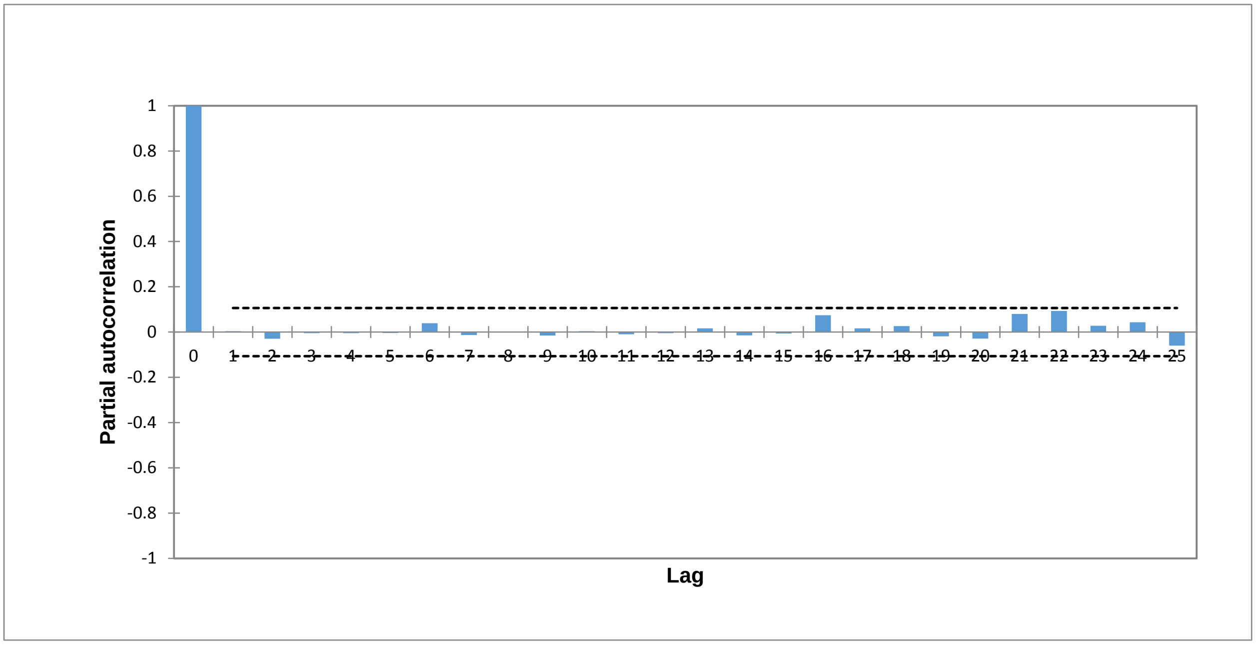


Figure 3.10: First differenced PACF Plot of Central Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017.

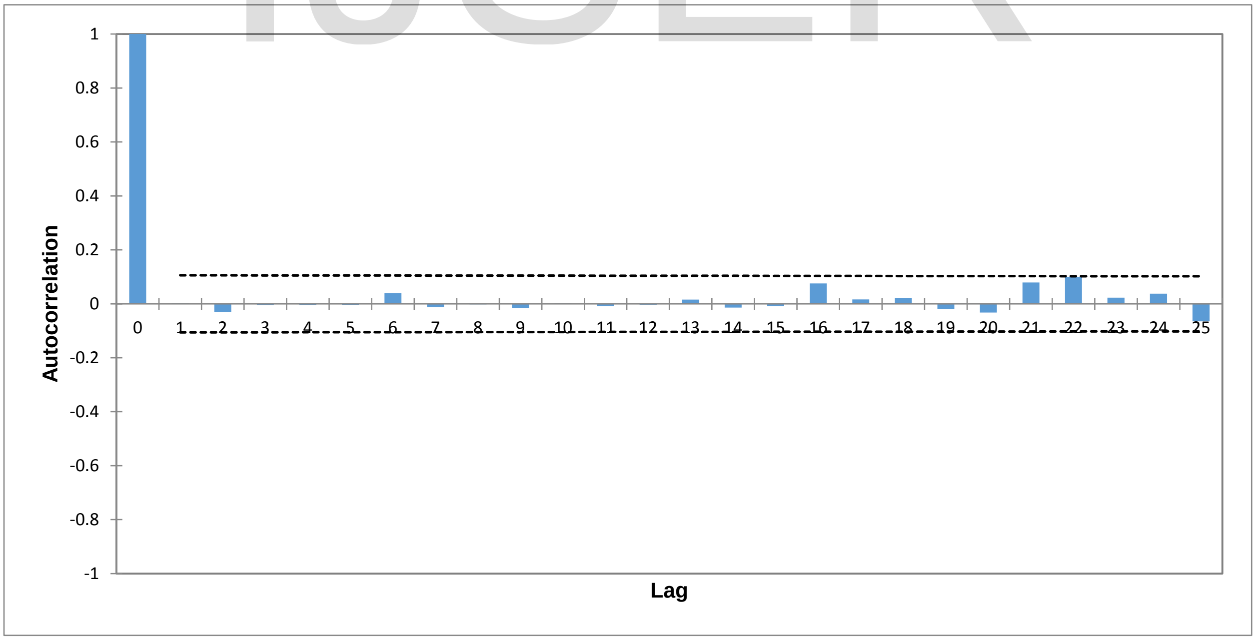


Figure 3.11: First differenced ACF Plot of Selling Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017.

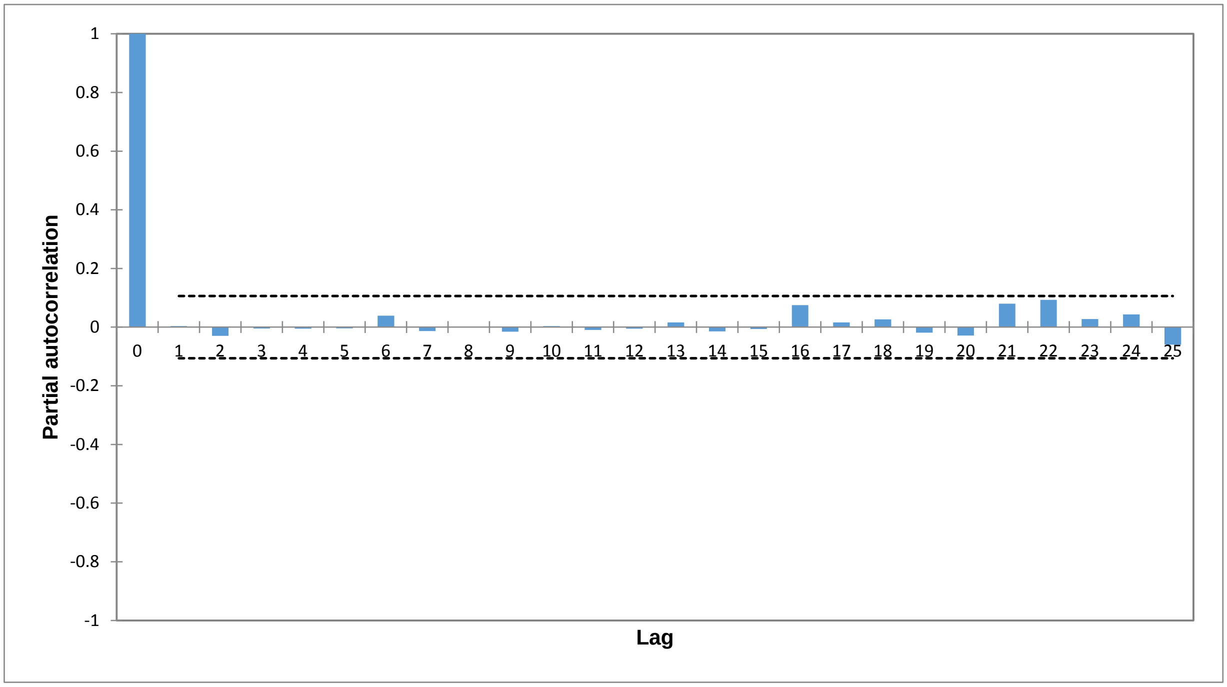


Figure 3.12: First differenced PACF Plot of Selling Rate of Daily Exchange Rate of Naira to US Dollar from January 2016 to May 2017.

The ACF for the three differenced series cuts off after $lag(0)$ and shows a slight spike at $lag(22)$. Similarly, the PACF for the three series tails off after $lag(0)$ and equally shows a spike at $lag(22)$, which implies that $ARIMA(1,1,1)$ model could be appropriate for the series. However, Given that ACF alternates in sign and has similar pattern with PACF, which indicate $MA(1)$ model, an $ARIMA(0,1,1)$ model could as well be appropriate for the series. Therefore, it is important to apply a model selection criterion in order to obtain best optimal and parsimonious model for the series. Therefore, Akaike Information Criterion (AIC) and Bayesian information criterion (BIC) were applied to select the best model. The results are shown in Table 3.5.

Table 3.5: ARIMA Models for the Buying, Central and Selling Rates of Exchange Rates of Naira to U. S. Dollar

	Buying Rate			Central Rate			Selling Rate		
ARIMA	P d q	AIC	BIC	P d q	AIC	BIC	P d q	AIC	BIC
ARIMA	0 1 0	0	0	0 1 0			0 1 0		
ARIMA	0 1 1	2039.604	2047.22	0 1 1	2040.308	2047.93	0 1 1	2039.69	2047.306
ARIMA	0 1 2	2041.624	2053.03	0 1 2	2042.13	2053.545	0 1 2	2041.71	2053.116
ARIMA	1 1 0	2177.59	2185.206	1 1 0	2040.309	2047.931	1 1 0	2177.689	2185.305
ARIMA	1 1 1	2041.625	2053.031	1 1 1	2042.273	2053.689	1 1 1	2041.711	2053.117
ARIMA	1 1 2	2043.603	2058.853	1 1 2	2044.177	2059.374	1 1 2	2043.749	2058.874
ARIMA	2 1 0	2137.479	2148.886	2 1 0	2042.129	2053.544	2 1 0	2137.573	2148.979
ARIMA	2 1 1	2043.43	2058.614	2 1 1	2044.177	2059.373	2 1 1	2043.517	2058.701
ARIMA	2 1 2	2045.523	2064.473	2 1 2	2046.319	2065.223	2 1 2	2045.577	2064.718

Based on the AIC and BIC information criteria, it was observed in Table 3.5 that the best model for the three series, buying rates, central rates and selling rates is $ARIMA(0,1,1)$ which had the minimum value for both AIC and BIC.

3.3 Estimates of the Parameters of the Identified Model

The maximum likelihood estimate of the parameters of the models for buying rate, central rate and selling rate were obtained with the aid of a Minitab statistical software. The results obtained are as given in tables 3.6, 3.7 and 3.8.

Table 3.6: Final Estimates of Parameters for Buying Rate model

Variable	**Coefficient**	**Standard error(SE)**	**T-Stat**	**Probability**
MA (1)	**-0.0040**	**0.0542**	**-0.07**	**0.941**
Constant	**0.3171**	**0.2618**	**1.21**	**0.227**

From table 3.6 the estimated model for buying exchange rate is,

$$Y_t = -0.004 y_{t-1} + \varepsilon_t$$

Table 3.7: Final Estimates of Parameters for Central Rate model

Variable	**Coefficient**	**Standard error(SE)**	**T-stat**	**Probability**
MA 1	**-0.0039**	**0.0542**	**-0.07**	**0.943**
Constant	**0.3171**	**0.2618**	**1.21**	**0.227**

From table 3.7, the estimated model for the central exchange rate is,

$$Y_t = -0.039 y_{t-1} + \varepsilon_t$$

Table 3.8: Final Estimates of Parameters for Selling Rate model

Variable	Coefficient	Standard error(SE)	T-Stat	Probability
MA (1)	-0.0040	0.0542	-0.07	0.941
Constant	0.3171s	0.2618	1.21	0.227

From table 3.8, the estimated model for the selling exchange rate model is

$$Y_t = -0.0040 y_{t-1} + \varepsilon_t$$

3.4 Results of Test for Heteroscedasticity

The ARCH-LM test for constant variance was carried out with the aid of XLSTAT software. The p-values obtained were 0.546, 0.545 and 0.546 respectively for buying, central and selling exchange rates. Each of the p-values was greater than the 0.05 level of significance, indicating homogeneity of the error variances for the three series.

3.5 Results of Test for Serial Auto-Correlation

The results of Ljung Box test for residuals autocorrelation at lags 12, 24, 36 and 48 respectively, gave p-values of 1.00, 0.979, 0.954 and 0.519 for buying rates which indicate the absence of serial correlation among the error terms since each of the p-values is greater than 0.05 level of significance.

From table (4.6), the coefficient (-0.0040) is less than 1, thus, $ARIMA(0,1,1)$ model is valid and stationary condition was met and satisfied. Therefore we conclude that $ARIMA(0,1,1)$ model for buying rate is adequate for forecast.

The results of Ljung Box test for residuals autocorrelation at lags 12, 24, 36 and 48 respectively, gave p-values of 1.00, 0.979, 0.954 and 0.520 for central rates which indicate the absence of serial correlation among the error terms since each of the p-values is greater than 0.05 level of significance.

From table 4.7, since the coefficient (- 0.0039) is less than 1, $ARIMA(0,1,1)$ model is valid and stationary condition was met and satisfied. Therefore, it can be concluded that the $ARIMA(0,1,1)$ model for the central exchange rates is adequate for forecast.

The results of Ljung Box test for residuals autocorrelation at lags 12, 24, 36 and 48 respectively, gave p-values of 1.00, 0.979, 0.954 and 0.519 for selling rates which indicate the absence of serial correlation among the error terms since each of the p-values is greater than 0.05 level of significance.

From table 4.8, since the coefficient (- 0.0040) is less than 1, $ARIMA(0,1,1)$ model is valid and stationary condition was met and satisfied. Therefore, it can be concluded that the $ARIMA(0,1,1)$ model for selling exchange rate is adequate for forecast.

3.6 Forecasting

Since the model is tested adequate, next is to make forecast using the various estimated model. The forecast Naira per US Dollar exchange rate for the period of (182 days) is represented with the first 32 rows in Tables, 4.25, 4.26, and 4.27 for Buying rate, Central rate, and Selling rate respectively.
The forecasting performance was illustrated in table 4.28 based on MAPE (mean absolute percentage error), MSE (mean square error), and RMSE (root mean square error).

ISSN 2229-5518

Table 4.25: Daily Exchange Rate Forecast of Buying Rate of Naira/US dollar

Forecasts from 3rd April 2017 to 19th October 2017 180days with 95% Confidence Limits						
Period	Date Rate	Forecast	Lower	Upper	Actual	Error value
312	4/3/2017	305.666	296.211	315.12	305.3	-0.366
313	4/4/2017	305.983	292.585	319.38	305.25	-0.733
314	4/5/2017	306.3	289.881	322.719	305.2	-1.1
315	4/6/2017	306.617	287.651	325.583	305.2	-1.417
316	4/7/2017	306.934	285.726	328.143	305.15	-1.784
317	4/10/2017	307.251	284.015	330.487	305.15	-2.101
318	4/11/2017	307.568	282.468	332.668	305.1	-2.468
319	4/12/2017	307.885	281.05	334.72	305.1	-2.785
320	4/13/2017	308.202	279.738	336.667	305.05	-3.152
321	4/18/2017	308.52	278.514	338.525	305	-3.52
322	4/19/2017	308.837	277.366	340.308	305	-3.837
323	4/20/2017	309.154	276.282	342.025	305	-4.154
324	4/21/2017	309.471	275.256	343.685	305	-4.471
325	4/24/2017	309.788	274.281	345.295	304.95	-4.838
326	4/25/2017	310.105	273.351	346.859	304.9	-5.205
327	4/26/2017	310.422	272.462	348.382	304.9	-5.522
328	4/27/2017	310.739	271.61	349.868	304.85	-5.889
329	4/28/2017	311.056	270.793	351.32	304.85	-6.206
330	5/2/2017	311.373	270.006	352.741	304.8	-6.573
331	5/3/2017	311.69	269.248	354.133	304.75	-6.94
332	5/4/2017	312.008	268.516	355.499	304.7	-7.308
333	5/5/2017	312.325	267.81	356.84	304.7	-7.625
334	5/8/2017	312.642	267.126	358.158	304.7	-7.942
335	5/9/2017	312.959	266.464	359.454	304.65	-8.309
336	5/10/2017	313.276	265.822	360.73	304.6	-8.676
337	5/11/2017	313.593	265.199	361.987	304.6	-8.993
338	5/12/2017	313.91	264.594	363.227	304.6	-9.31
339	5/15/2017	314.227	264.006	364.449	304.6	-9.627
340	5/16/2017	314.544	263.434	365.655	304.45	-10.094
341	5/17/2017	314.861	262.877	366.846	304.45	-10.411
342	5/18/2017	315.179	262.334	368.023	304.45	-10.729
343	5/19/2017	315.496	261.806	369.186	304.45	-11.046

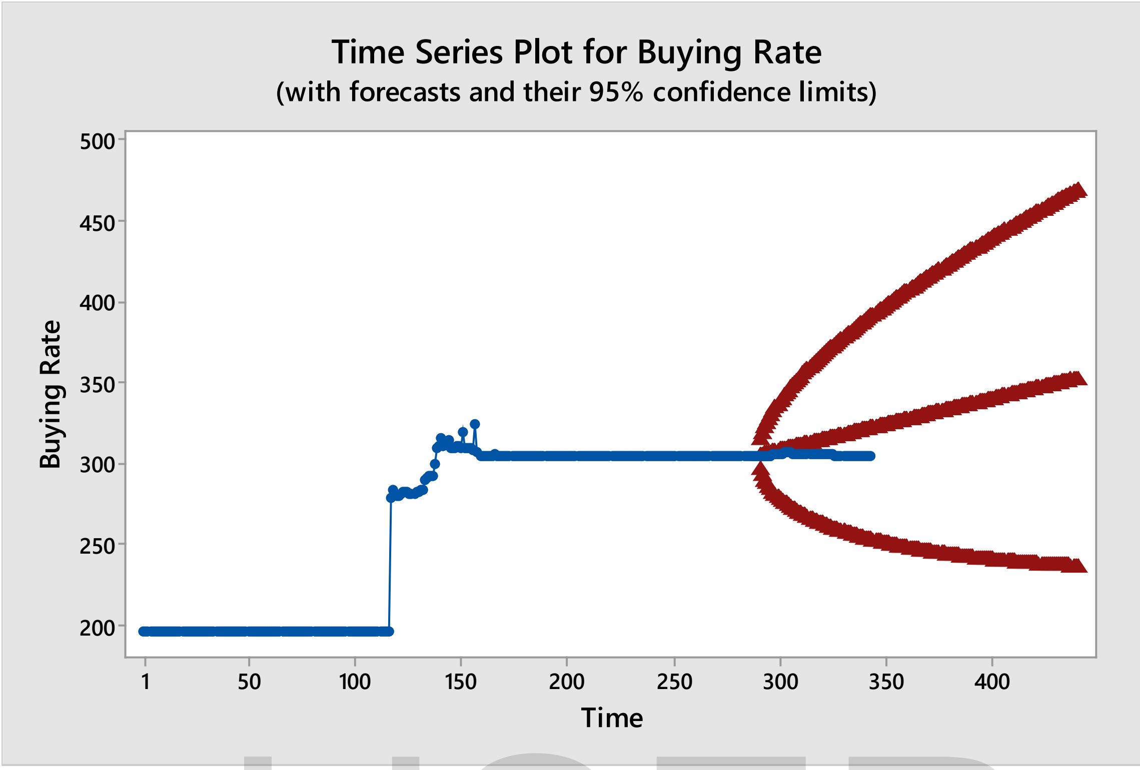


Figure 4.22: 95% confidence limits forecast for the (Buying Rate) of Daily exchange rate of naira to US dollar from January 2016 to May 2017.

From figure 4.22 we observed that, the (Buying Rate) naira will depreciate more within the forecasted period of (April to October 2017) (150 days).

Table 4.26: Daily Exchange Rate Forecast on Central Rate of Naira/ US dollar

Forecasts from 3rd April 2017 to 19th October 2017 180days with 95% Confidence Limits						
Period_1	Date Rate	Forecast Rate	Lower	Upper	Actual Rate	Error value
312	4/3/2017	306.166	296.71	315.621	305.8	-0.366
313	4/4/2017	306.483	293.085	319.881	305.75	-0.733
314	4/5/2017	306.8	290.38	323.22	305.7	-1.1
315	4/6/2017	307.117	288.151	326.083	305.7	-1.417
316	4/7/2017	307.434	286.225	328.643	305.65	-1.784
317	4/10/2017	307.751	284.515	330.987	305.65	-2.101
318	4/11/2017	308.068	282.968	333.169	305.6	-2.468
319	4/12/2017	308.385	281.55	335.221	305.6	-2.785
320	4/13/2017	308.702	280.238	337.167	305.55	-3.152
321	4/18/2017	309.02	279.014	339.025	305.5	-3.52
322	4/19/2017	309.337	277.865	340.808	305.5	-3.837
323	4/20/2017	309.654	276.782	342.525	305.5	-4.154
324	4/21/2017	309.971	275.756	344.186	305.5	-4.471
325	4/24/2017	310.288	274.781	345.795	305.45	-4.838
326	4/25/2017	310.605	273.851	347.359	305.4	-5.205
327	4/26/2017	310.922	272.962	348.882	305.4	-5.522
328	4/27/2017	311.239	272.111	350.368	305.35	-5.889
329	4/28/2017	311.556	271.293	351.82	305.35	-6.206
330	5/2/2017	311.873	270.506	353.241	305.3	-6.573
331	5/3/2017	312.191	269.748	354.633	305.25	-6.941
332	5/4/2017	312.508	269.017	355.999	305.2	-7.308
333	5/5/2017	312.825	268.31	357.34	305.2	-7.625
334	5/8/2017	313.142	267.626	358.658	305.2	-7.942
335	5/9/2017	313.459	266.964	359.954	305.15	-8.309
336	5/10/2017	313.776	266.322	361.23	305.1	-8.676
337	5/11/2017	314.093	265.699	362.487	305.1	-8.993
338	5/12/2017	314.41	265.094	363.726	305.1	-9.31
339	5/15/2017	314.727	264.506	364.949	305.1	-9.627
340	5/16/2017	315.044	263.934	366.155	304.95	-10.094
341	5/17/2017	315.362	263.377	367.346	304.95	-10.412
342	5/18/2017	315.679	262.834	368.523	304.95	-10.729
343	5/19/2017	315.996	262.306	369.686	304.95	-11.046

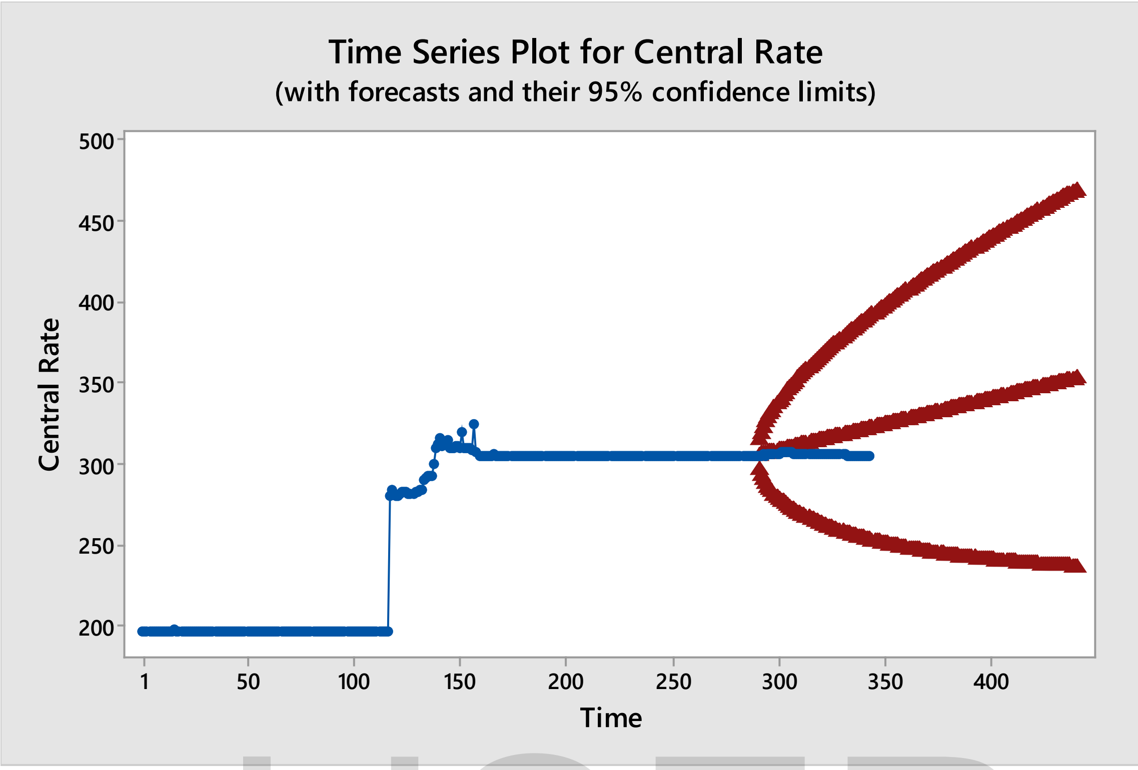
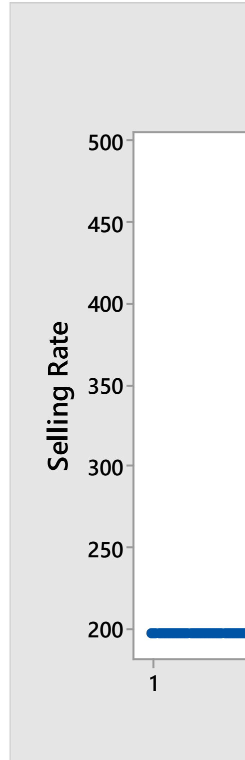


Figure 4.23: 95% confidence limits forecast for the (Central Rate) of Daily exchange rate of naira to US dollar from January 2016 to May 2017.

From figure 4.23 we observed that, the (central Rate) of naira will depreciate more within the forecasted period of (3rd of April to 15th of October 2017) (150 days).

Table 4.27: Daily Exchange Rate Forecast on Selling Rate of Naira/US-Dollar



Forecasts from 3rd April 2017 to 19th October 2017 180days with 95% Confidence Limits						
Period_2	Date Rate	Forecast Rate	Lower	Upper	Actual_Rate	Error value
312	4/3/2017	306.666	297.211	316.12	306.3	-0.366
313	4/4/2017	306.983	293.585	320.38	306.25	-0.733
314	4/5/2017	307.3	290.881	323.719	306.2	-1.1
315	4/6/2017	307.617	288.651	326.583	306.2	-1.417
316	4/7/2017	307.934	286.726	329.143	306.15	-1.784
317	4/10/2017	308.251	285.015	331.487	306.15	-2.101
318	4/11/2017	308.568	283.468	333.668	306.1	-2.468
319	4/12/2017	308.885	282.05	335.72	306.1	-2.785
320	4/13/2017	309.202	280.738	337.667	306.05	-3.152
321	4/18/2017	309.52	279.514	339.525	306	-3.52
322	4/19/2017		278.366	341.308	306	306
323	4/20/2017	310.154	277.282	343.025	306	-4.154
324	4/21/2017	310.471	276.256	344.685	306	-4.471
325	4/24/2017	310.788	275.281	346.295	305.95	-4.838
326	4/25/2017	311.105	274.351	347.859	305.9	-5.205
327	4/26/2017	311.422	273.462	349.382	305.9	-5.522
328	4/27/2017	311.739	272.61	350.868	305.85	-5.889
329	4/28/2017	312.056	271.793	352.32	305.85	-6.206
330	5/2/2017	312.373	271.006	353.741	305.8	-6.573
331	5/3/2017	312.69	270.248	355.133	305.75	-6.94
332	5/4/2017	313.008	269.516	356.499	305.7	-7.308
333	5/5/2017	313.325	268.81	357.84	305.7	-7.625
334	5/8/2017	313.642	268.126	359.158	305.7	-7.942
335	5/9/2017	313.959	267.464	360.454	305.65	-8.309
336	5/10/2017	314.276	266.822	361.73	305.6	-8.676
337	5/11/2017	314.593	266.199	362.987	305.6	-8.993
338	5/12/2017	314.91	265.594	364.227	305.6	-9.31
339	5/15/2017	315.227	265.006	365.449	305.6	-9.627
340	5/16/2017	315.544	264.434	366.655	305.45	-10.094
341	5/17/2017	315.861	263.877	367.846	305.45	-10.411
342	5/18/2017	316.179	263.334	369.023	305.45	-10.729
343	5/19/2017	316.496	262.806	370.186	305.45	-11.046

Figure 4.24: 95% confidence limits forecast for the (Selling Rate) of Daily exchange rate of naira to US dollar from January 2016 to May 2017.

From figure 4.24 we observed that, the (selling Rate) of naira will depreciate more within the forecasted period of April to October, 2017 (182 days).

Table 4.28: In-sample forecast error performance of ARIMA (0, 1, 1) model.

	Buy Rate	Central Rate	Selling Rate
No. observation	32	32	32
ME	-5.72284375	-5.72290625	-5.71325
MSE	42.86713159	42.86821609	42.79645444
RMSE	6.547299565	6.547382385	6.54189991
MAPE	1.878084331	1.875028178	1.868805906

Table 4.28, revealed that the Selling rate model has the least MSE, RMSE MAPE and ME values. Thus, buying rate model outperformed the central rate and the buying rate model during the forecast period. Hence it was concluded that the model for the selling rate model is the best performing model on the basis of these results.

4. **Conclusion**

In this paper, Box and Jenkins approach was used to model the daily exchange rate of Nigerian Naira for U. S. Dollar for Buying rate, Central rate and Selling rate. The series for the three rates were found to be non-stationary but were observed to be stationary after first order differencing. It was observed that ARIMA (0, 1, 1) with the same constant value (0.3171) for the three series was the most appropriate model for Buying rates, Central rates and Selling rates. Tests for heteroscedasticity and serial correlation were performed which showed homogeneity of the variances and absence of serial correlation among the error terms. The models were used to make forecast for 180 days which showed a good fit of the models but it was observed that the ARIMA (0,1,1) fitted the Selling series most.

Acknowledgement: We appreciate the comments of Prof. (Mrs.) Joy Nwabueze, and Nduka, Uchenna Chinedu on this paper. We also appreciate the authors whose papers improved this manuscript and the anonymous referees.

References

Akaike, H. A (1974) A new look at the statistical model identification. A Paper Presented At The Nigerian British Chamber Of Commerce, Transactions on Automatic Control, **Vol.19**, No.16; 723.

Awogbeni C. and Alagbe S. (2011). Empirical Modelling of Nigerian Exchange Rate Volatility, Mathematical Theory and Modeling, Vol. 1(3), 2224-5804.

Box, G.E.P and Jenkins G. M, (1976). *Time series Analysis*; Forecasting and Control, Holden – Day Inc., USA.

David, R. O., Dikko H. G. and Gulumbe S. U. (2016). Modeling Volatility of the Exchange Rate of the Naira to Major Currencies, C. B. N. Journal of Applied Statistics, Vol. 7(2).

Emenike K. O. and Ali P. I. (2016). Test for Autoregressive Conditional Heteroscedastcity in Naira/US Dollar Exchange Rate in Nigeria, International Journal of Economic and Financial Management, Vol. 4(1), 11-16.

Esam M. (2017). "*Time Series Analysis – part 2*". Islamic University of Gaza - Department of Mathematics.

Makridakis, S., Wheelwright S.C. and Hyndman R.J., (1998). Forecasting Methods and Applications, 3rd Edn., John Wiley, New York.

Mohammed and Abdulmuahymin (2016). Modeling the Exchange Ability of Nigerian Currency (Naira) with Respect to US Dollar, International Journal of Scientific and Engineering Research, Vol.7 (7).

Nwogu, E.C and Iwu H.C (2010) ARIMA model of crude oil exports in Nigeria. African Journals online, 17; 285-292

Oladejo, M .O, and Abdullahi .B (2013), A Suitable Model for the Forecast of Exchange Rate in Nigeria (Nigerian Naira versus US Dollar) Department of mathematics Nigerian Defense Academy P.M.B 2109 Kaduna State.

Olatunji, O. M. and Bello A. (2013). A Suitable Model for the Forecast of Exchange Rate in Nigeria (Nigeria Naira Versus US Dollar), International Journal of Science and Research, 2319-7064.

Onasanya, O.K and Adeniji, O. E (2013). Forecasting of Exchange Rate Between Naira and US Dollar Using Time Domain Model. International Journal of Development and Economic Sustainability, Vol. 1(1), 45-55.

Tasi'u, M., Yakubu, M. and Gulumba S. U. (2014). Exchange Rate Volatility of Nigeria Naira against some Major Currencies in the World: An Application of Multivariate GARCH Models, International Journal of Mathematics and Statistics Invention, Vol. 2(6), 52-65.